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Innovative Approaches to Modeling Economic Dynamics: Taylor Matrix Methods in Solving Budget Constraint Differential Equations

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Abstract

In recent times, the consensus has emerged that a profound understanding of mathematics is essential to excel as an economist, leading to a rapid increase in the number of articles incorporating mathematical methodologies within the field of economics. This study explores the dynamic aspects of economic modeling by focusing on the budget constraint equation of a group, treating it as a partial differential equation. The budget constraint problem is a common economic challenge where individuals or entities face limitations on consumption choices due to financial constraints. We introduce an alternative method for solving this equation, employing a matrix-based approach grounded in collocation points and Taylor polynomials. This technique streamlines the resolution process, transforming the solution of the group's budget constraint differential equation into a system of matrix equations featuring unknown Taylor coefficients. The paper contributes to the ongoing discourse on mathematical models in economics by presenting an innovative methodology that enhances economists' analytical toolkit for understanding and navigating the intricate dynamics of economic systems. The proposed method is then applied to the budget constraint equation of the group, providing a systematic and efficient approach to obtain numerical solutions. The study includes a numerical example to illustrate the application of the technique, demonstrating its efficiency, precision, and ability to yield highly accurate approximations for the budget constraint equation. The results highlight the versatility and effectiveness of the Taylor matrix-collocation techniques in solving economic problems.

Keywords: Mathematical economics, economics-mathematics relationship, group's budget constraint differential equation

1. Introduction

In the contemporary view of economic research, the interdependent relationship between mathematics and economics has become increasingly evident. A deep understanding of mathematical principles is widely recognized as crucial for achieving excellence in the field of economics, necessitating further exploration and integration of mathematical methodologies through ongoing research. Among the diverse mathematical tools at economists' disposal, differential equations have emerged as powerful instruments for modeling and analyzing various complex systems. Traditionally associated with physical and engineering problems, these equations have found substantial applications in economics, risk theory, and various social sciences. Differential equations, especially partial differential equations (PDEs), serve as powerful tools for modeling, analyzing, and addressing a wide range of physical and engineering problems. As a crucial branch of applied mathematics, they find application in diverse fields such as network design, fluid dynamics, wave motion, telecommunications, electromagnetic wave distribution, and electronic dynamics. Beyond engineering and physical systems, these equations play a pivotal role in economics, risk theory, and numerous social sciences. In recent years, the development of numerical methods has provided effective solutions for solving these equations. On the other hand, some researchers have criticized the use of mathematical models in the relationship between mathematics and economics. Beed and Kane (1991) provided an overview of the varied and scattered criticisms directed at the mathematization of economics over the last seventy years, highlighting limited comprehensive reviews of the arguments against the emphasis on mathematics in contemporary economics. Bilgin (2006) critically examined the role of mathematical methods in economics, weighing arguments that emphasize the clarity and precision of mathematical approaches against concerns about the potential for unfruitful outcomes in economic theory due to over-reliance on math without a solid theoretical foundation. Aydın (2016) suggested that the relationship between economics and mathematics, particularly within the context of neoclassical economic theory, has been a subject of criticism. Kaleci and Buluş (2016) explored the historical context of the integration of mathematics into economics, examining both positive and negative criticisms, highlighting the current significance of mathematics in economics, and investigating its specific applications within the field. Yücel (2022) advocated that an overdependence on mathematical models, leading to a detachment of economics from social realities, signifies a precarious and perilous situation.

In literature, many researchers have explored diverse aspects of mathematical modeling in economics, ranging from continuous-time heterogeneous agent models and PDE-based approaches in socio-economic processes to the impact of knowledge on wealth evolution and challenges associated with optimal control problems in economic growth theory. Achdou et al. (2014) reviewed the literature on continuous-time versions of heterogeneous agent models in macroeconomics, emphasizing the common mathematical structure involving coupled nonlinear PDEs, particularly the Hamilton–Jacobi–Bellman equation for optimal control and an equation governing the distribution of individual state variables in the population, known as a mean field game, shedding light on the existing knowledge and identifying avenues for future research. Burger et al. (2014) highlighted the growing significance of PDE-based approaches in modeling socio-economic processes, anticipating that this emerging field will become a focal point in PDE-centered modeling, presenting new mathematical challenges and tools while emphasizing the importance of understanding socio-economic complexity for addressing future challenges. Pareschi and Toscani (2014) introduced and explored a modified nonlinear kinetic equation of Boltzmann type, originally proposed by Cordier et al. (2005), to depict the impact of knowledge on wealth evolution within a system of interacting agents engaged in binary trades. Boucekkine et al. (2013) reviewed the application of parabolic PDEs in economic growth theory, highlighting the challenges in solving optimal control problems with infinite time horizons and emphasizing the ill-posedness problem. Brito (2020) reviewed and exemplified the use of differential equations in macroeconomic models.

Our study delves into the realm of economic dynamics by focusing on the budget constraint equation of a group, treating it as a PDE. A budget constraint problem typically arises in economics and refers to the limitation imposed on an individual or entity's consumption choices by the amount of income or resources available. The fundamental idea is that individuals, households, or firms face constraints on their spending or investment decisions due to their limited financial resources. The budget constraint is often expressed in the form of an equation that represents the relationship between income, prices of goods and services, and the quantity of those goods and services consumed or purchased. Mathematically, it can be represented by an equation which indicates that the total income must be allocated among different goods or services, each with its own price and quantity. The challenge is to optimize the allocation to maximize utility, satisfaction, or profit, subject to the budget constraint. In the context of mathematical modeling, budget constraints are often formulated as optimization problems to find the optimal combination of goods or services that an individual or entity can afford given their budget limitations. Moon and Silver (2000) studied a multi-item newsvendor problem with a budget constraint on replenishment quantities, explicitly considering fixed costs for non-zero replenishments, presenting dynamic programming procedures for cases where demand distributions are assumed known. Vairaktarakis (2000) introduced an alternative approach to stochastic optimization for the multi-item newsboy model with a budget constraint and demand uncertainty, utilizing integer programming models with minimax objectives. Numerous studies (Karabakal et al., 2000; Guo et al.,2017; Şahin et al., 2010; Chekuri and Kumar, 2004) have investigated various aspects of budget constraint problems in the literature.

This paper introduces an alternative method for solving this equation, employing a matrix-based approach grounded in collocation points and Taylor polynomials. The proposed technique streamlines the resolution process, transforming the solution of the group's budget constraint differential equation into a system of matrix equations featuring unknown Taylor coefficients. Then an explanation is presented using Taylor polynomials, providing a new way to approach economic challenges. Through this exploration, the paper contribute to the ongoing discourse on mathematical models in economics, presenting an innovative methodology that enhances the analytical toolkit available to economists for understanding and navigating the intricate dynamics of economic systems.

Many researchers have primarily used Taylor polynomials, along with other types of polynomials, to solve both differential and integral equations. Kurt and Sezer (2008) introduced a practical matrix method for approximating solutions to high-order linear Fredholm integro-differential equations with constant coefficients under initial-boundary conditions, utilizing Taylor polynomials. Kurt and Çevik (2008) proposed a straightforward numerical method for solving the single degree of freedom system using Taylor polynomials in matrix form, allowing for the determination of both particular and general solutions of the differential equation. Biçer and Yalçınbaş (2016) introduced an approximate method using Bernoulli polynomials to solve hyperbolic partial differential equations, demonstrating its validity through matrix transformation, employing a Bernoulli coefficients matrix as the unknown, conducting error analysis based on residual function, and providing illustrative examples for method accuracy. Gümgüm et al. (2018) introduced a numerical matrix-collocation technique utilizing Lucas polynomials and standard/Chebyshev-Lobatto collocation points to address functional integro-differential equations with variable delays and initial conditions, and demonstrated its practicability through illustrative examples. Bayku and Sezer (2017) introduced a collocation method based on hybrid Taylor and Lucas polynomials, for solving a higher-order linear nonhomogeneous pantograph-type delay differential equation with variable coefficients and delays. Erdem Biçer and Yalçınbaş (2019) employed the Bernoulli collocation method to suggest an approximate solution for the telegraph equation. Elmacı et al. (2022) introduced a matrix-collocation method employing Euler polynomials to approximate solutions for singularly perturbed two-point boundary-value problems, presenting a systematic approach involving algebraic equations, coefficient determination. Çayan et al. (2022) proposed the Laguerre matrix collocation method, based on orthogonal Laguerre polynomials, aiming to decrease computational costs in mathematical models by directly utilizing Laguerre polynomials without transforming them into the truncated Taylor polynomial basis. Savaşaneril (2023) introduced a new and straightforward numerical method for solving the free vibration of a single degree of freedom system using Lucas polynomials in matrix form, allowing for the determination of particular and general solutions. Recently, Çevik and Sezer (2023) provided a brief history of polynomial matrix method, mentioned the types of polynomials utilized in collocation approach, explained its fundamental principles and surveyed engineering applications.

2. The Budget Constraint Equation

In this study, we will explore the budget constraint problem of the group proposed by Brito (2020). Let $w(x,t)$ represent the financial wealth of an individual aged x at time t. Then, the budget constraint equation is given in Eq. (1).

$$\frac{\partial w(x,t)}{\partial t}+\frac{\partial w(x,t)}{\partial x}=s(x,t)+r\,w(x,t) \tag{1}$$

The general solution of Eq. (1), where $s(x,t)$ is the accumulation of an individual at age x and time t and r is the interest rate, is given in Eq. (2).

$$w(x,t)=\left(\int_0^x s(z,z-x+t)e^{-rz}dz+f(t-x)\right)e^{rx} \tag{2}$$

If the initial (at birth) accumulation is given by $w(0,t)=0$ and the accumulation function is $s(a,t)=e^{ba(K-a)+gt}-c$, then the solution function is given in Eq. (3).

$$w(a,t)=\frac{\sqrt{\pi}}{2\sqrt{b}}\left(\phi\left(\frac{Kb+g-r}{2\sqrt{b}}\right)-\phi\left(\frac{(K-2a)b+g-r}{2\sqrt{b}}\right)\right)e^{\frac{K^2b^2\left(\left(2K+4(t-a)\right)g-2(K-2a)r\right)b+(g-r)^2)}{4b}} \\ -\frac{c}{r}(1-e^{ra}). \tag{3}$$

In this study, we introduce a Taylor Collocation Method based on matrix equations (Kurt and Sezer, 2008) for the solution of the equation given in Eq. (1). It is given in Eq. (6).

$$w(x,t)\cong w_N(x,t)=\sum_{m=0}^{N}\sum_{n=0}^{N}a_{mn}x^m\,t^n \tag{4}$$

The objective here is to obtain an approximate solution using this approach, and the unknowns in this solution are represented by the Taylor coefficients denoted as a_{mn}; $m,n=1,\dots,N$.

3. Fundamental Relations

To obtain the numerical solution of the group's budget constraint equation using the Taylor Collocation Method, we first compute the unknown function's Taylor coefficients. For this purpose, the solution function of Eq. (1) can be written in matrix form as Eq. (5).

$$w_N(x,t)=\mathbf{X}(x)\overline{\mathbf{X}}(t)\mathbf{A} \tag{5}$$

where $X(x)$, $\overline{X}(t)$, A are presented in Eq. (6), Eq. (7), Eq. (8), respectively.

$$\mathbf{X}(x)=\begin{bmatrix}1 & x & x^2 & \text{K} & x^N\end{bmatrix} \tag{6}$$

$$\overline{\mathbf{X}}(t)=diag\begin{bmatrix}\mathbf{X}(t) & \mathbf{X}(t) & \mathbf{X}(t) & \text{K} & \mathbf{X}(t)\end{bmatrix} \tag{7}$$

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_0 & \mathbf{A}_1 & \text{L} & \mathbf{A}_N\end{bmatrix}^T, \tag{8}$$

Derivatives of w with respect to x and t are given in Eq. (9) and Eq. (10), respectively.

$$\frac{\partial w}{\partial x}=\mathbf{X}(x)\mathbf{B}\overline{\mathbf{X}}(t)\mathbf{A} \tag{9}$$

$$\frac{\partial w}{\partial t}=\mathbf{X}(x)\overline{\mathbf{X}}(t)\overline{\mathbf{B}}\mathbf{A}, \tag{10}$$

Where $B, \overline{B}$ are presented in Eq. (11), Eq. (12), respectively.

$$\mathbf{B}=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 2 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & 0 & N\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}, \tag{11}$$

$$\overline{\mathbf{B}}=diag\left[\mathbf{B}\right]_{(N+1)^2 x (N+1)^2} \tag{12}$$

4. Solution Methodology

At this stage of our analysis, we are composed to express the foundational matrix equation corresponding to Eq. (1). This crucial step involves synthesizing the pertinent elements and relationships inherent in our study to construct a comprehensive matrix equation. For this purpose, we express matrix Eq. (5), Eq. (9), and Eq. (10) in relation to Eq. (1), incorporating essential adjustments, resulting in Eq. (13):

$$\mathbf{X}(a)\overline{\mathbf{X}}(t)\overline{\mathbf{B}}\mathbf{A}+\mathbf{X}(a)\mathbf{B}\overline{\mathbf{X}}(t)\mathbf{A} = s(a,t)+r\mathbf{X}(a)\overline{\mathbf{X}}(t) \tag{13}$$

With additional modifications, we reach the following Eq. (14):

$$\mathbf{W}=\left\{\mathbf{X}(a)\overline{\mathbf{X}}(t)\overline{\mathbf{B}}+\mathbf{X}(a)\mathbf{B}\overline{\mathbf{X}}(t) - r\mathbf{X}(a)\overline{\mathbf{X}}(t)\right\} \tag{14}$$

As a result, we acquire Eq. (15):

$$\mathbf{W}(\mathrm{x,t})\mathbf{A}=\mathbf{G}(x,t). \tag{15}$$

The standard or Chebyshev-Lobatto collocation points used in matrix system are given in Eq. (16), respectively, such that x_i and t_i are the Chebyshev-Lobatto collocation points (Gümgüm et al., 2018) on Ω defined to be:

$$x_i=\frac{a+b}{2}+\frac{a-b}{2}\cos\left(\frac{\pi i}{N}\right), \quad t_j=\frac{c+d}{2}+\frac{c-d}{2}\cos\left(\frac{\pi j}{N}\right) \quad i,j=0,1,\ldots,N. \tag{16}$$

where $i,j=0,1,\ldots,N$, $a=x_0<x_1<\ldots<x_N=b$, $c=t_0<t_1<\ldots<t_N=d$,

By substituting collocation points into Eq. (14), we can express Eq. (16) as follows in Eq. (17):

$$\mathbf{W}(\mathrm{x}_i,\mathrm{t}_j)=\left\{\mathbf{X}(x_i)\overline{\mathbf{X}}\left(t_j\right)\overline{\mathbf{B}}-\alpha^2\mathbf{X}(x_i)\mathbf{B}^2\overline{\mathbf{X}}\left(t_j\right)\right\} \tag{17}$$

At this point, it is possible to formulate the fundamental matrix equation corresponding to Eq. (1) in the following form in Eq. (18):

$$\mathbf{WA}=\mathbf{G} \text{ or } \left[\mathbf{W};\mathbf{G}\right] \tag{18}$$

Here, $W(x_i, t_j)$, A, G are presented in Eq. (19), Eq. (20) and Eq. (21), respectively.

$$\mathbf{W}(x_i,t_j)=\left[\mathbf{W}_0 \quad \mathbf{W}_1 \quad \mathbf{W}_2 \quad \mathrm{L} \quad \mathbf{W}_N\right]^T \tag{19}$$

$$\mathbf{A}=\left[a_{00} \quad a_{01} \quad \mathrm{L} \quad a_{0N} \quad a_{10} \quad \mathrm{L} \quad a_{1N} \quad a_{N0} \quad \mathrm{L} \quad a_{NN}\right]^T_{1x(+1)^2} \tag{20}$$

$$\mathbf{G}=\left[g(x_0,t_0) \quad g(x_0,t_0) \quad \mathrm{L} \quad g(x_0,t_1) \quad \mathrm{L} \quad g(x_0,t_N) \quad \mathrm{L} \quad g(x_N,t_N)\right]^T_{1x(+1)^2} \tag{21}$$

5. Matrix Representation of Conditions

The initial condition of the Eq. (1) is given as in Eq. (22):

$$\mathbf{W}(0,\mathrm{t})=0 \tag{22}$$

The matrix form of Eq. (22) can be as follows in Eq. (23):

$$\mathbf{W}(0,\mathrm{t})=\mathbf{X}(0)\overline{\mathbf{X}}(\mathrm{t}_i)\mathbf{A}=0, \quad i=0,1,\ldots,N \tag{23}$$

Thus, we obtain Eq. (24).

$$\mathbf{UA}=\mathbf{0} \text{ or } \left[\mathbf{U};\mathbf{0}\right] \tag{24}$$

Now, we shall express Eq. (1) in the form of Eq. (3) and then use that form to determine the Taylor coefficients $a_{m,n}; m,n=1,2,\ldots,N$ for the approximate solution provided in Eq. (19). This is achieved by replacing any row of Eq. (18) with Eq. (24), leading to the creation of the following augmented matrix Eq. (25).

$$\left[\widetilde{\boldsymbol{W}};\widetilde{\boldsymbol{G}}\right]=\begin{bmatrix}\boldsymbol{W} & ; & \boldsymbol{G}\\ \boldsymbol{U} & ; & \mathbf{0}\end{bmatrix} \tag{25}$$

The solution to this system of matrix equations provides the Taylor coefficients for the solution function in Eq. (3).

6. Numerical Application

To illustrate the application of the discussed approach, we will now delve into a numerical example addressing the problem given by Brito (2020).

$$K=88, \quad b=0.0029, \quad r=0.02, \quad g=0.02, \quad c=50.$$

In accordance with our proposed method for addressing the problem under the specified initial conditions, we utilize collocation points derived from Eq. (16) with *N=5*. Upon computing the approximate solution, the resulting solution function is as follows:

$$
\begin{aligned}
e_{10} = {} & 894.2290902552949a - 16.92928912951453\,a^2 + 0.7431029209505327\,a^3 \\
& - 0.005142068770569089a^4 + 0.000023867421496233018a^5 - 134.882793421843a\,t + \\
& + 66.895884054600454a^2\,t - 0.17570865993074752a^3\,t + 0.001708040778020449a^4\,t \\
& - 6.178284282641527.\,10^{-6}a^5\,t + 4.3709040726463755a\,t^2 - 0.2662943416850872a^2\,t^2 \\
& + 0.006881340967762259a^3\,t^2 - 0.000072502781801472667a^4\,t^2 + 2.725177925228526.\,10^{-7}a^5\,t^2 \\
& - 0.06564194413920374a\,t^3 + 0.004515561642674559a^2\,t^3 - 0.00011887445378840342a^3\,t^3 \\
& + 1.291677975687328 2.\,10^{-6}a^4t^3 - 4.964727260568466.\,10^{-9}a^5\,t^3 + 0.0004426138874452158a\,t^4 \\
& - 0.000033096690905968 11a^2\,t^4 + 8.988316952126963.\,10^{-7}a^3\,t^4 - 1.0041283977949947.\,10^{-8}a^4\,t^4 \\
& + 3.945102910652574.\,10^{-11}a^5t^4 - 1.1526531471275705.\,10^{-6}a\,t^5 + 9.200076204012123.\,10^{-8}a^2\,t^5 \\
& - 2.5605523746328023.\,10^{-9}a^3\,t^5 + 2.915848456262097.\,10^{-11}a^4\,t^5 - 1.1639536893915662.\,10^{-13}a^5\,t^5
\end{aligned}
$$

Figure 1 illustrates the above numerical result obtained through the proposed technique.

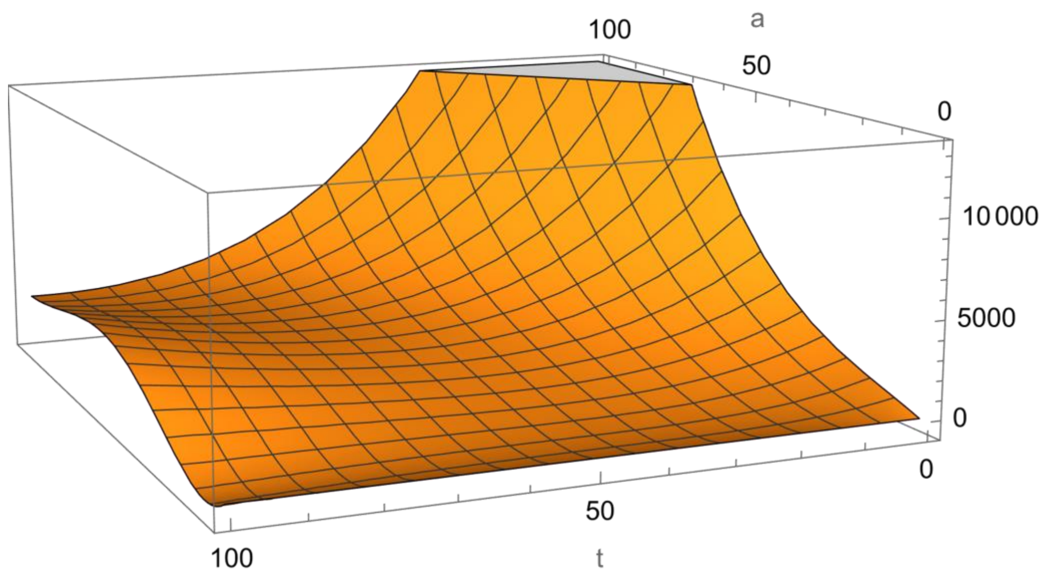

Figure 1. The illustration of the numerical result for the budget constraint differential equation

As can be understood from the results, the individual tends to be a clear debtor in young age and a creditor in old age.

7. Conclusions

We have introduced a novel approach utilizing Taylor matrix-collocation techniques to address the budget constraint equation of the group. This methodology facilitates the seamless transformation of the group's budget constraint equation, including initial conditions, into a matrix system based on algebraic functions. The efficacy and accuracy of this innovative technique are demonstrated through its application to an illustrative example. The results not only showcase the efficiency and precision of our approach but also reveal its capability to yield highly accurate approximations for the budget constraint equation. This study underscores the high accuracy, rapid convergence, and excellent approximation capabilities of the new approach employing Taylor matrix-collocation techniques in solving the budget constraint equation for a group.

Author Statement

The author confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

Conflict of Interest

The author declares no conflict of interest.

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